Kinetic theory of the non-local electrodynamic response in anisotropic metals: skin effect and application to $PdCoO_2$

D. Valentinis[1,2], G. Baker[3,4], T. W. Branch[3,4], J. Day[3,4], M. Oudah[3], P. McGuinness[5], S. Khim[5], P. Surówka[6,7,8], R. Moessner[9], A. P. Mackenzie[5,10], D. A. Bonn[3,4], and J. Schmalian[1,2]

[1] *Institut für Theorie der Kondensierten Materie, Karlsruher Institut für Technologie, 76131 Karlsruhe, Germany*

[2] *Institut für Quantenmaterialien und Technologien, Karlsruher Institut für Technologie, 76131 Karlsruhe, Germany*

[3] *Stewart Blusson Quantum Matter Institute, University of British Columbia, Vancouver, British Columbia, Canada V6T 1Z4*

[4] *Department of Physics and Astronomy, University of British Columbia, Vancouver, British Columbia, Canada V6T 1Z1*

[5] *Max Planck Institute for Chemical Physics of Solids, Nöthnitzer Straße 40, 01187 Dresden, Germany*

[6] *Department of Theoretical Physics, Wroc law University of Science and Technology, 50-370 Wrocław, Poland*

[7] *Institute for Theoretical Physics, University of Amsterdam, 1090 GL Amsterdam, The Netherlands*

[8] *Dutch Institute for Emergent Phenomena (DIEP), University of Amsterdam, 1090 GL Amsterdam, The Netherlands*

[9] *Max Planck Institute for the Physics of Complex Systems, Nöthnitzer Straße 38, 01187 Dresden, Germany and*

[10] *Scottish Universities Physics Alliance, School of Physics and Astronomy, University of St Andrews, St Andrews KY16 9SS, United Kingdom*

Electronic motion in new-generation ultra-pure materials, with extremely long electronic mean free paths, deviates from the usual diffusive response, with striking consequences like viscous[1–5] and ballistic[6–8] flow. Although such regimes show in DC transport [9–14], their fingerprints on finite-frequency electrodynamic properties have long remained elusive [4, 15–17]. The theory of electrodynamics in ultra-pure metals faces two additional challenges: the Fermi surface is often spatially anisotropic, and spatial correlations between moving electrons (non-locality) are non-negligible; these properties are at odds with established models of metallic conduction, which assume a local response and spherically symmetric Fermi surfaces. In this work, we construct a kinetic theory of non-local electrodynamics, based on the Boltzmann equation, in two and three dimensions and for an arbitrary Fermi-surface shape [18]. We focus on the skin effect and the surface impedance as probes of non-local optical regimes for polygonal Fermi surfaces.

We solve the Boltzmann equation using the collision-operator formalism, which allows us to find the electronic distribution function for an arbitrarily shaped Fermi surface [18]. We include momentum-relaxing and momentum-conserving scattering with characteristic rates. Focusing on the hexagonal and square polygonal geometries, we explicitly find the optical conductivity and compare it to the isotropic case. We then connect to the basic observables of the skin depth and the surface impedance, which describe the damping of electromagnetic fields in the interior of a metal (skin effect). This way, we construct the "skin-effect phase diagrams" for the surface impedance, characterizing quantitatively the crossovers between the hydrodynamic, relaxational, and ballistic regimes. We find that these crossovers are qualitatively affected by the anisotropy of the Fermi surface, and by its orientation with respect to the applied field. Large flat segments where the electron velocity is parallel to the field can completely suppress the non-locality of the optical response.

The theory is compared with a new technique developed for broadband microwave spectroscopy of the delafossite metal $PdCoO_2$. The surface resistance for three distinct sample geometries shows differing power laws in ballistic regime, which proves the occurrence of anisotropic skin effect in $PdCoO_2$ [19]. Our work permits a novel symmetry-based diagnostic for non-local electrodynamics.

---

[1] R. N. Gurzhi, Soviet Physics Uspekhi **11**, 255 (1968).
[2] S. Conti and G. Vignale, Phys. Rev. B **60**, 7966 (1999).
[3] I. V. Tokatly and O. Pankratov, Phys. Rev. B **62**, 2759 (2000).
[4] D. Forcella, J. Zaanen, D. Valentinis, and D. van der Marel, Phys. Rev. B **90**, 456 (2014).
[5] P. Ledwith, H. Guo, A. Shytov, and L. Levitov, Phys. Rev. Lett. **123**, 116601 (2019).
[6] G. E. H. Reuter and E. H. Sondheimer, Proc. R. Soc. A **195** (1948), 10.1098/rspa.1948.0123.
[7] E. H. Sondheimer, Adv. Phys. **50**, 499 (2001).
[8] J. M. Link, D. E. Sheehy, B. N. Narozhny, and J. Schmalian, Phys. Rev. B **98**, 195103 (2018).
[9] M. J. M. de Jong and L. W. Molenkamp, Phys. Rev. B **51**, 13389 (1995).

[10] J. Crossno, J. K. Shi, K. Wang, X. Liu, A. Harzheim, A. Lucas, S. Sachdev, P. Kim, T. Taniguchi, K. Watanabe, T. A. Ohki, and K. C. Fong, Science **351**, 1058 (2016).
[11] P. J. Moll, P. Kushwaha, N. Nandi, B. Schmidt, and A. P. Mackenzie, Science **351**, 1061 (2016).
[12] J. A. Sulpizio, L. Ella, A. Rozen, J. Birkbeck, D. J. Perello, D. Dutta, M. Ben-Shalom, T. Taniguchi, K. Watanabe, T. Holder, R. Queiroz, A. Principi, A. Stern, T. Scaffidi, A. K. Geim, and S. Ilani, Nature **576**, 75 (2019).
[13] P. H. McGuinness, E. Zhakina, M. König, M. D. Bachmann, C. Putzke, P. J. W. Moll, S. Khim, and A. P. Mackenzie, Proc. Natl. Acad. Sci. U.S.A. **118** (2021).
[14] M. D. Bachmann, A. L. Sharpe, A. W. Barnard, C. Putzke, T. Scaffidi, N. Nandi, S. Khim, M. Koenig, D. G. Gordon, A. P. Mackenzie, and P. J. W. Moll, arXiv:2103.01332 (2021).
[15] P. Gallagher, C.-S. Yang, T. Lyu, F. Tian, R. Kou, H. Zhang, K. Watanabe, T. Taniguchi, and F. Wang, Science **364**, 158 (2019).
[16] D. Valentinis, Phys. Rev. Research **3**, 023076 (2021).
[17] D. Valentinis, J. Zaanen, and D. Van Der Marel, Sci. Rep. **11**, 1 (2021).
[18] G. Baker, T. W. Branch, J. Day, D. Valentinis, M. Oudah, P. McGuinness, S. Khim, P. Surówka, R. Moessner, J. Schmalian, A. P. Mackenzie, and D. Bonn, arXiv:2204.14239 (2022).
[19] G. Baker, T. W. Branch, J. Day, D. Valentinis, M. Oudah, P. McGuinness, S. Khim, P. Surówka, R. Moessner, J. Schmalian, A. P. Mackenzie, and D. Bonn, arXiv:2204.14239 (2022).

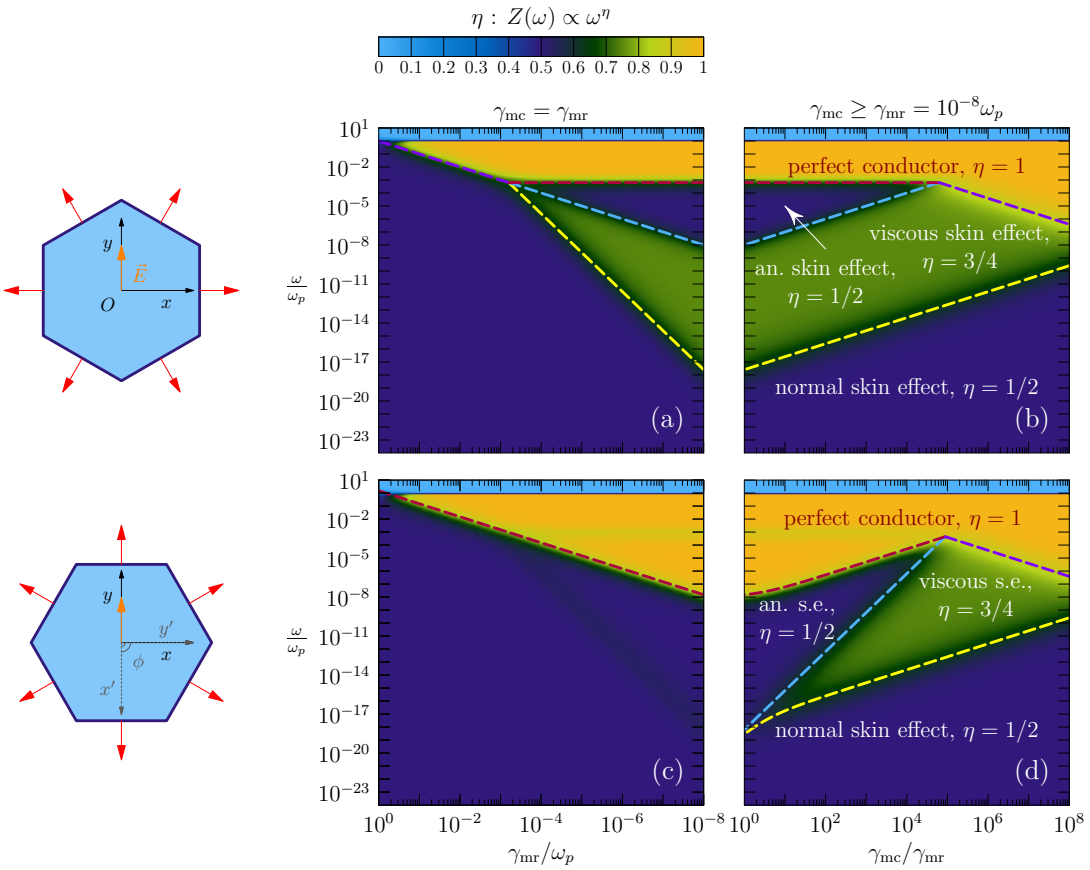

FIG. 1. Orientational dependence of skin effect regimes for a hexagonal Fermi surface, as measured by the surface impedance $Z(\omega)$ as a function of relaxation rate γ_{mr}, momentum-conserving collision rate γ_{mr}, and frequency ω/ω_p, where ω_p is the plasma frequency. The normal (diffusive), anomalous (ballistic), and viscous skin effect regimes are detected through the frequency scaling of $Z(\omega)$. The differences between panels (a) and (b) exemplify the anisotropic nature of skin effect in anisotropic metals, as explained by kinetic theory [18].